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Comparison of Sometimes Pool Estimation Procedures of Error Variance Using One Preliminary Test in a Mixed Anova Model With Another Using Two Preliminary Tests

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SUMMARY

A comparison of estimation procedures involving oné preliminary test of significance (PTS) with another involving two preliminary tests for the estimation of the true error variance in a analysis of variance mixed model situation is presented. The bias and mean square error of a sometimes pool estimation (STPE) procedure using one PTS has been obtained and its relative efficiency over never pool estimation (NPE) procedure has been compared with the results of another STPE procedure using two PTS which has been studied separately.

Key words : PTS, Sometimes pool estimation (STPE), Never pool estimation (NPE).

Introduction

The proposed study pertains to a comparison of two conditionally specified inference procedures for which detailed bibliography may be seen in Bancroft and Han [2] and Han, Rao and Ravichandran [3].

1.1 Application

The present study relates to a experimental design model for a split plot in a time experiment in which some of the factors are fixed and the remaining random. These experiments are analogous to usual split plot experiments and are characterised mainly by the feature that observations made are on the same whole unit over a period of time. Such situations arise frequently in experiments of forage crops (Steel and Torrie [5]) or with perennial and semi-perennial plants such as orchard and plantation crops like sugarcane, bananas, tropical fodder grasses etc. Considering a mixed model situation, one is interested in an estimator of the error variance when uncertainties regarding the parameters involved in the model specification exist.

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1.2 Problem to be solved

Ali and Srivastava [1] considered the following conditionally specified mixed ANOVA model corresponding to above mentioned split plot in time experiment having frequent use in forage crops.

$$y_{ijk} = \mu + \alpha_i + \beta_j + \delta_{ij} + \tau_k + (\alpha\tau)_{ij} + (\beta\tau)_{jk} + \epsilon_{ijk}$$

where Y_{ijk} = yield on the k^{th} cutting of the j^{th} variety in the i^{th} block, $i = 1, 2; ..., r; j = 1, 2, ..., s; k = 1, 2, ..., t$; μ is the true mean effect, α_i is the random block effect and β_j, τ_k,* are the fixed effects of varieties and cuttings respectively. The cuttings effect, i.e. τ_k, is of main interest for which the abridged ANOVA table is given as

Table 1. Mixed model abridged ANOVA for a split-plot in the experiment –

Source of variation	Degrees of freedom	Mean squares	Expected mean squares
Treatments (Cuttings)	$n_4 = t - 1$	V_4	$\sigma_4^2 = \sigma_\epsilon^2 + s\,\sigma_{\alpha\tau}^2 + r\,s[\sigma_\tau^2]$
			$= \sigma_3^2 (1 + 2\lambda_4/n_4)$
True Error (Cuttings × Block)	$n_3 = (t-1)(r-1)$	V_3	$\sigma_3^2 = \sigma_\epsilon^2 + s\sigma_{\alpha\tau}^2$
Doubtful Error II (Cuttings × Varieties)	$n_2 = (t-1)(s-1)$	V_2	$\sigma_2^2 = \sigma_\epsilon^2 + r[\sigma_{\beta\tau}^2]$
			$= \sigma_1^2 (1 + 2\lambda_2/n_2)$
Doubtful Error I (Cuttings × Variety × Block)	$n_1 = (t-1)(s-1)(r-1)$	V_1	$\sigma_1^2 = \sigma_\epsilon^2$

In Table 1, λ_2 and λ_4 are the non-centrality parameters. It may be noted that model (1.1) applies to any three-way cross classification lay out where any two factors may be fixed effects and the third being random.

The problem to be solved here is to find an estimator of σ_3^2, the true error variance, pertaining to the estimation situatioin when (i) the cutting × variety interaction already exists, i.e., $\sigma_{\beta\tau}^2 > 0$ (or $\sigma_2^2 > \sigma_1^2$) ; e.g. in case of forage crops usually different varieties respond differently to different cuttings; and (ii) the doubtful situation is that $\sigma_{\alpha\tau}^2$ may be equal to zero. The other estimation situation aries out of the test proposed by Ali *et. al., where the doubtful conditions are that* $(\alpha\tau)_{ik}$ *and/or* $(\beta\tau)_{jk}$ *may equal to zero. i.e.* $\sigma_{\alpha\tau}^2$ *may be equal to zero (see Table 1). In other words, the former situation corresponds to only one* *doubtful condition* $\sigma_3^2 \geq \sigma_1^2$ *while the latter to two doubtful conditions* σ_3^2 *and/or* $\sigma_2^2 \geq \sigma_1^2$. *In general, the assumptions corresponding to usual (never pool) estimator* V_3 of σ_3^2 is $\sigma_3^2 \neq \sigma_2^2 \neq \sigma_1^2$.

The present paper is concerned only with the first estimation situation. The estimation for the second situation has been studied separately (Singh *et. al.*, [4], whose results will be used here for the sake of comparison. The doubtful condition existing in the first estimation situation is resolved by performing the preliminary test $H_{01} : \sigma_3^2 = \sigma_1^2$ (i.e. $\theta_{31} = 1.0$) vs $H_{11} : \sigma_3^2 > \sigma_1^2$ $(\theta_{31} > 1.0)$ based on which the final test of treatment differences is made in another study by testing $H_0 : \sigma_4^2 = \sigma_3^2$ (i.e. $\lambda_4 = 0$) against $H_1 : \sigma_4^2 > \sigma_3^2$ (i.e. $\lambda_4 > 0$), where σ_4^2 is the true treatment variance. In this study the same preliminiary test is used in the estimation procedure for estimating σ_3^2. The estimation situation arising out of the doubtful conditions was resolved by the preliminiary tests $H_{01} : \sigma_3^2 = \sigma_1^2$ (i.e. $\theta_{31} = 1.0$) vs $H_{31} : \sigma_3^2 > \sigma_1^2$ $(\theta_{31} > 1.0)$ and $H_{02} : \sigma_2^2 = \sigma_1^2$ (i.e. $\lambda_2 = 0$) vs $H_{12} : \sigma_2^2 > \sigma_1^2$ (i.e. $\lambda_2 > 0$) in succession (Singh *et. al.*, [4], based on the outcomes of which Ali and Srivastava finally tested H_0 vs H_1.

Thus, using the similar sometimes pool procedure as adopted by Ali *et. al.* and Singh *et. al.*, a sometimes pool estimator V* for estimating σ_3^2 corresponding to the above mentioned first estimation situation is proposed as follows :

$$V^* = \begin{cases} V_3 & \text{if } V_3/V_1 \geq F(n_3, n_1 ; \sigma_1) \\ V_{13} & \text{if (i) } V_3/V_1 < F(n_3, n_1 ; \alpha_1) \\ & \text{and (ii) } V_2/V_{13} \geq F(n_2, n_{13} ; \alpha_2) \end{cases} \quad (1.2)$$

The estimaor V corresponding to second estimation situation as studied by Singh *et. al.* [4] is

$$V = \begin{cases} V_3 & \text{if } V_3/V_1 \geq F(n_3, n_1 ; \alpha_1) \\ V_{13} & \text{if (i) } V_3/v_1 < F(n_3, n_1; \alpha_1) \\ & \text{and (ii) } V_2/V_{13} \geq F(n_2, N_{13}; \alpha_2) \\ V_{123} & \text{if (i) } V_3/v_1 < F(n_3, n_1; \alpha_1) \\ & \text{and (ii) } V_2/V_{13} < F(n_2, N_{13}; \alpha_2) \end{cases} \quad (1.3)$$

where

$V_{13} = (n_1 V_1 + n_3 V_3)/(n_1 + n_3)$, $V_{123} = (n_1 V_1 + n_2 V_2 + n_3 V_3)/(n_1 + n_2 + n_3)$

and $F(n_i, n_j; \alpha_k)$ is the upper 100 α_k % point of the central F-distribution with (n_i, n_j) degrees of freedom.

In this paper we study the bias, mean square error and relative efficiency of V* with respect to V_3 and compare their numerical results with those of V extracted from Singh *et. al.* [4].

1.3 The motivation for proposing V* :

The motivation behind proposing V* is that, usually different varieties of forage crops respond differently to different cuttings, which indicates the prior existence of cuttings × variety interaction i.e., $\sigma^2_{\beta\tau} > 0$ $(\sigma_2^2 > \sigma_1^2)$. In this case, when we have the doubtful situation $\sigma_3^2 \geq \sigma_1^2$ then it is likely that one preliminary test estimator V* may be more appropriate for estimating the error variance $\sigma_3{}^2$ than V since the latter is an estimator meant for the more general parametric situation σ_3^2 and /or $\sigma_2^2 \geq \sigma_1^2$. Therefore, comparision of both the specific situation estimator V* and the general situation estimator V vis-a-vis the usual estimator V3., which corresponds to the situation $\sigma_3^2 \neq \sigma_2^2, \sigma_1^2$, has also been made.

2. *Mean Value, Bias and Mean Square Error of Estimator V* along with its Efficiency relative to never pool Estimator V_3*

The mean value E(V*) of estimaor V* is given by

$$E(V^*) = E\ V_3 \mid V_3/V_1 \geq F(n_3, n_1; \alpha_1)]\ \Pr[V_3/V_1 \geq F(n_3, n_1; \alpha_1)]$$
$$+ E[V_{13} \mid V_3/V_1 < F(n_3, n_1; \alpha_1)]\ \Pr[V_3/V_1 < F(n_3, n_1; \alpha_1)] \quad (2.1\ a)$$

or, say $$E(V^*) = E_1^* P_1^* + E_2^* P_2^* \quad (2.1\ b)$$

where $$E_1^* = E[V_3 \mid V_3/V_1 \geq F(n_3, n_1; \alpha_1)],$$

$$P_1^* = \Pr[V_3/V_1 \geq F(n_3, n_1; \alpha_1)]$$

and $E_2^* P_2^*$ is similarly defined.

For maintaining the continuity of presentation the derivations for $E_1^* P_1^*$, $E_2^* P_2^*$ and $E_3^* P_3^*$ have been relegated to the appendix. The expressions derived there are substituted in (2.1) to get the mean value E (V*). Then the bias is obtained by BIAS(V*) = E (V*) − σ_3^2.

The mean square error of the estimator V^* is defined as

$$MSE(V^*) = E(V^* - \sigma_3^2]^2 = E(V^{*2}) - 2\sigma_3^2 E(V^*) + (\sigma_3^2)^2 \quad (2.2)$$

In the r.h.s. of equation (2.2) the only unevaluated quantity is $E(V^{*2})$, given σ_3^2. Therefore, to evaluate $E(V^{*2})$ we can express it as in case of $E(V^*)$ given by (2.1). Thus,

$$E(V^{*2}) = E_{11}^* P_1^* + E_{22}^* P_2^* \quad (2.3)$$

where

$$E_{11}^* = E[(V_3^2 \mid (V_3/V_1) \geq F(n_3, n_1; \alpha_1)],$$

$$P_1^* = Pr[(V_3/V_1) \geq F(n_3, n_1; \alpha_1)]$$

and $E_{22}^* P_2^*$ is similarly defined. The dervied results from the appendix are used in (2.3) to get the exression for $E(V^{*2})$. Then $MSE(V^*)$ is evaluated from (2.2) using the final expressions for $E(V^{*2})$ and $E(V^*)$.

The relative efficiency of the estimator V^* with respect to the never pool estimator V_3 is given by $R.E. = MSE(V_3)/MSE(V^*) = \{2(\sigma_3^2)^2/n_3\}/MSE(V^*)$, since $MSE(V_3) = E(V_3^2) - (\sigma_3^2)^2$, $E(V_3^2) = (\sigma_3^2)^2 + \{2(\sigma_3^2)^2/n_3\}$.

3. *Discussion of Results*

In order to facilitate the comparison of estimators V^* and V we have considered the three sets of degrees of freedom $n_1 = 2, n_3 = 2$; $n_1 = 2, n_3 = 4$ and $n_1 = 10, n_3 = 2$ for calculating the results of V^* corresponding to the three sets $n_1 = 2$, $n_2 = 2$, $n_3 = 2$; $n_1 = 2$, $n_2 = 2$, $n_3 = 4$ and $n_1 = 10, n_2 = 10, n_3 = 2$ for V, whose reasults were extracted from Singh *et. al.* [4]. Three values of preliminary levels of significance were considered, i.e. $\alpha_1 = \alpha_2 = \alpha_p = 0.50, 0.25$ and 0.05, for numerical investigation. However, the first choice, $\alpha_1 = \alpha_2 = \alpha_p = 0.50$, was found to be most suitable from the point of view of relative efficiency. This was further confirmed by the same choice of α obtained in a separate study of Singh *et. al.* [4]. Therefore, for brevity, the tables for $\alpha_1 = \alpha_2 = \alpha_p = 0.50$ have only been given. The results of V^* and V have been combined and presented in Tables A.1 to A.3 of the appendix.

3.1. Bias

A perusal of columns fifth and sixth of Tables A.1 through A.3 reveals that the bias of V^* is negative for all sets of degrees of freedom and for all the values of θ_{31}. However, with the increase in the variance ratio θ_{31}, the numerical value (i.e. ignoring sign) of bias goes on increasing.

Now, if we compare Tables A.1 with A.3 and Tables A.1 with A.2, we find that the numerical bias of V^* increases as n_1 increases from 2 to 10 or n_3 increases from 2 to 4 for different fixed values of θ_{31}.

To compare the bias of V^* with that of V, the entries against BIAS of V corresponding to $\lambda_2 > 0$ have been considred, which arises from the assumption that $\sigma^2_{\beta\tau} > 0$ or $\sigma^2_2 > \sigma^2_1$. Thus, on comparing the corresponding entries we find that the bias of V^* is always negative and that of V is always positive. However, on making numerical comparison (ignoring sign), it is found that the bias of V^* is numerically less than that of V for $\theta_{31} = 1$ when λ_2 is moderate to high. For $\theta_{31} > 1$ the bias of V^* is numerically more than that of V except when n_1, n_3 are very small, λ_2 is high and θ_{31} is moderate where V^* is again less biased.

3.2 Mean squre error and relative efficiency

The entries for the mean square errors and relative efficiency have been presented in the columns seven to eleven of Tables A.1 through A.3. Since the effect of mean square error of V^* manifests itself through its relative efficiency (RE) over V_3, the numerical discussion will be confined to the RE only.

It can be seen from Tables A.1 to A.3 that for all sets of degrees of freedom under study, the relative efficiecy, e (V^*, V_3) %, decreases with the increase of variance ratio θ_{31} in its entire range of values considered except $\theta_{31} = 1.0$.

For a given value of θ_{31}, an increase in the true error degrees of freedom n_3 decreases the relative efficiency of V^* with respect of V_3 (see last columns of Tables A.1 and A.2) This might be due to smaller decrease in MSE (V^*) compared to the decrease in MSE (V_3) as n_3 increases from 2 to 4. However, the efficiencies of both the estimators V^* and V3 almost remain the same with the increase in the doubtful error d.f. n_1 as this increase causes a negligible decrease in e (V^*, V_3) %. It may be noted that the increase in n_2 does not matter in case of V^*.

As regards the comparision of the estimators V^* V, the entries of $e(V^*, V_3)$ % and $e(V, V_3)$ % in Tables A.1 to A.3 are compared in the same way as in case of bias. It is observed that for $\theta_{31} = 1$, the values of $e(V^*, V_3)$ % are higher than those of $e(V, V_3)$ % for moderate to high values of λ_2. For $\theta_{31} > 1$, the values of $e(V^*, V_3)$ % are found to less than those of $e(V, V_3)$ % except the cases when n_1, n_3 are very small, θ_{31} is moderate and λ_2 is high for which case the values of $e(V^*, V_3)$ % are again higher.

4. *Conclusion*

When the interaction $(\beta\tau)_{jk}$, say, cuttings × varieties, already exists, e.g., when different varieties of forage crops respond differently to different cuttings, then the proposed estimator V^* should be preferred under following situqations. For a situation when the variance of true error seems to be almost equal to that of the first doubtful error, V^* is preferable to both V and V_3 on account of better efficiency. Similarly, when the degrees of freedom for the true error (n_3) is at premium, say less than 2 irrespective of those of first doubtful error (n_1), then also the estimator V^* should be preferred to both V and V_3. In addition to this, when the true error variance seems to be moderately higher than the first doubtful error variance, the second doubtful error variance is higher than the first doubtful error variance, and the degrees of freedom available for these errors are very few, say less than or equal to 2, then again V^* is preferable to V.

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APPENDIX

A.1 Joint density function :

The joint density of V_1 and V_3 is given by

$$f(V_1, V_3) = A\, V_1^{\frac{1}{2}n_1 - 1}\, V_3^{\frac{1}{2}n_3 - 1} \exp\left[-\tfrac{1}{2}\{n_1 V_1/\sigma_1^2 + n_3 V_3/\sigma_3^2\}\right] \quad \text{(A.1 a)}$$

where
$$A = \frac{(n_1/\sigma_1^2)^{\frac{1}{2}n_1}(n_3/\sigma_3^2)^{\frac{1}{2}n_3}}{2^{\frac{1}{2}(n_1+n_3)}\Gamma(\frac{1}{2}n_1)\Gamma(\frac{1}{2}n_3)} \quad \text{(A.1 b)}$$

Introducing the transformations

$$u_1 = n_3 V_3/(n_1 V_1 \theta_{31}),\ u_2 = n_1 V_1/(2\sigma_1^2) \quad \text{(A.2)}$$

where $0 \le u_1 < \infty, 0 \le u_2 < \infty; \theta_{31} = \sigma_3^2/\sigma_1^2$ the joint density function can be rewritten as

$$f(u_1, u_2) = A_1\, u_1^{\frac{1}{2}n_3 - 1}\, u_2^{\frac{1}{2}(n_1+n_3)-1} \exp\left[-\tfrac{1}{2}\{2u_2(1+u_1)\}\right] \quad \text{(A.3a)}$$

where
$$A_1 = \frac{1}{\Gamma(\frac{1}{2}n_1)\Gamma(\frac{1}{2}n_3)}, \text{ and } 0 \le u_1 < \infty, 0 \le u_2 < \infty \quad \text{(A.3b)}$$

A.2 Derivation of $E_1^*P_1^*$, $E_2^*P_2^*$ and $E_3^*P_3^*$:

To derive $E_1^* P_1^*$ we express V_3 and $\{(V_3/V_1) \ge F(n_3, n_1 : \alpha_1)\}$ in terms of u's, so that

$$E_1^* P_1^* = E\{(2\sigma_3^2/n_3)\, u_1 u_2 \mid u_1 \ge a\} \Pr(u_1 \ge a)$$
$$= \int_{u_1=a}^{\infty} \int_{u_2=0}^{\infty} (2\sigma_3^2/n_3)\, u_1 u_2\, f(u_1, u_2)\, du_2\, du_1 \quad \text{(A.4a)}$$

where $a = u_1^0/\theta_{31}$, $u_1^0 = (n_3/n_1)\, F(n_3, n_1; \alpha_1)$ (A.4b)

Then we apply the transformations

$$z = u_2(1+u_1), \text{ so that } u_2 = \frac{z}{1+u_1},\ du_2 = \frac{dz}{1+u_1}$$

and
$$y = \frac{1}{1+u_1}, \text{ so that } u_1 = \frac{(1-y)}{y},\ du_1 = -\frac{dy}{y^2} \quad \text{(A.5)}$$

in succession to integrate out u_2 and u_1. Then, we get

$$E_1^* P_1^* = A_1 \frac{2\sigma_3{}^2}{n_3} \Gamma\left(\tfrac{1}{2}(n_1+n_3)+1\right) B\, x_1\left(\tfrac{1}{2}n_1, \tfrac{1}{2}n_3+1\right) = A_2\, Bx_1\left(\tfrac{1}{2}n_1, \tfrac{1}{2}n_3+1\right) \qquad \text{(A.6 a)}$$

where $$A_2 = \frac{2\sigma_3{}^2}{n_3} \frac{\Gamma\left(\tfrac{1}{2}(n_1+n_3)+1\right)}{\Gamma\left(\tfrac{1}{2}n_1\right)\Gamma\left(\tfrac{1}{2}n_3\right)} \text{ and } x_1 = \frac{1}{1+a} \qquad \text{(A.6 b)}$$

We can also show that (A.6 a) reduces to

$$E_1^* P_1^* = \sigma_3^2\, Ix_1\left(\tfrac{1}{2}n_1, \tfrac{1}{2}n_3+1\right), \text{ where } I_x\ B_x(p,q)/B(p,q) \qquad \text{(A. 6d)}$$

The expression for $E_2^* P_2^*$ has been obtained in the similar way and is given below :

$$E_2{}^* P_2{}^* = A_2\left[Bx_2\left(\tfrac{1}{2}n_3, \tfrac{1}{2}n_1+1\right) + \theta_{31}\, Bx_2\left(\tfrac{1}{2}n_3+1, \tfrac{1}{2}n_1\right)\right] \qquad \text{(A.7 a)}$$

where A_2 is as given in (A.6 b) and using (A.6 b),

$$x_2 = a/(1+a) = 1 - \{1/(1+a)\} = 1 - x_1 \qquad \text{(A.7 b)}$$

A.3 Derivation of $E_{11}P_1$, $E_{22}P_2$ and $E_{33}P_3$:

Using similar procedures as in the evaluation of $E_1^* P_1^*$, $E_2^* P_2^*$ we can also evaluate $E_{11}^* P_1^*$, E_{22}^*, P_2^*. For the sake of brevity only the final espressions are given below :

$$E_{11}^* P_1^* = A_1\left\{4(\sigma_3^2/n_3)^2\right\}\Gamma\left(\tfrac{1}{2}(n_1+n_3)+2\right) Bx_1\left(\tfrac{1}{2}n_1, \tfrac{1}{2}n_3+2\right)$$

$$= \left\{(\sigma_3^2)^2 + 2(\sigma_3^2)^2/n_3\right\} Ix_1\left(\tfrac{1}{2}n_1, \tfrac{1}{2}n_3+2\right), \qquad \text{(A.8 a)}$$

$$E_{22}^* P_2^* = A_5\left[Bx_2\left(\tfrac{1}{2}n_3, \tfrac{1}{2}n_1+2\right) + 2\,\theta_{31}\, Bx_2\left(\tfrac{1}{2}n_3+1, \tfrac{1}{2}n_1+1\right)\right.$$

$$\left. + \theta_{31}^2\, Bx_2\left(\tfrac{1}{2}n_3+2, \tfrac{1}{2}n_1\right)\right] \qquad \text{(A. 8b)}$$

where $$A_5 = \frac{4(\sigma_1^2)^2}{(n_1+n_3)^2} \frac{\Gamma\left(\tfrac{1}{2}(n_1+n_3)+2\right)}{\Gamma\left(\tfrac{1}{2}n_1\right)\Gamma\left(\tfrac{1}{2}n_3\right)} \qquad \text{(A. 8c)}$$

A.4 Tables of numerical results :

Comparison of Bias, MSE of the sometimes pool estimators V^* and V, and their relative efficiencies over the never pool estimator V_3

Table A.1

$n_1 = n_2 = n_3 = 2, \ \alpha_1 = \alpha_2 = \alpha_p = 0.50$

θ_{31}	σ_1^2	σ_3^2	λ_2	BIAS		MSE		MSE (V_3)	REL. EFF.	
				V	V^*	V	V^*	V_3	$e(V,V_3)\%$	$e(V^*,V_3)\%$
1.0	1	1.0	0.00000	0.16696	—	0.97082	—	1.00000	103.0058	—
			2.41421	0.69085	–0.37500	1.78986	1.00000	1.00000	55.8704	100.0000
			4.44949	0.99158	–0.37500	2.51095	1.00000	1.00000	39.8256	100.0000
			6.46410	1.32735	–0.37500	3.95628	1.00000	1.00000	25.2763	100.0000
			8.47214	1.66204	–0.37500	5.84551	1.00000	1.00000	17.1072	100.0000
			10.47723	1.99616	-0.37500	8.17916	1.00000	1.00000	12.2262	100.0000
1.5	1	1.5	0.00000	0.12698	—	2.10762	—	2.25000	106.7553	—
			2.41421	0.56634	–0.65000	2.34918	2.17000	2.25000	95.7780	103.6866
			4.44949	0.77993	–0.65000	2.77126	2.17000	2.25000	81.1905	103.6866
			6.46410	1.04855	–0.65000	3.69471	2.17000	2.25000	60.8979	103.6866
			8.47214	1.31631	–0.65000	4.97408	2.17000	2.25000	45.2345	103.6866
			10.47723	1.58359	–0.65000	6.60940	2.17000	2.25000	34.0424	103.6866
2.0	1	2.0	0.00000	0.10149	—	3.78497	—	4.00000	105.6811	—
			2.41421	0.47471	–0.91667	3.62924	3.86111	4.00000	110.2159	103.5971
			4.44949	0.64254	–0.91667	3.84649	3.86111	4.00000	103.9909	103.5971
			6.46410	0.86639	–0.91667	4.41206	3.86111	4.00000	90.6607	103.5971
			8.47214	1.08954	–0.91667	5.27496	3.86111	4.00000	75.8300	103.5971
			10.47723	1.31225	–0.91667	6.43486	3.86111	4.00000	62.1614	103.5971

Contd ....

3.0	1	3.0	0.00000	0.07162	—	8.70203	—	9.00000	103.4241	—	
			2.41421	0.35561	–1.43750	8.03677	8.78125	9.00000	111.9853	102.4911	
			4.44949	0.47496	–1.43750	7.98965	8.78125	9.00000	112.6457	102.4911	
			6.46410	0.64285	–1.43750	8.09664	8.78125	9.00000	111.1572	102.4911	
			8.47214	0.81024	–1.43750	8.42784	8.78125	9.00000	106.7889	102.4911	
			10.47723	0.97724	–1.43750	8.98247	8.78125	9.00000	100.1951	102.4911	
5.0	1	5.0	0.00000	0.04449	—	24.63239	—	25.00000	101.4924	—	
			2.41421	0.23506	–2.45833	23.44346	24.69444	25.00000	106.6395	101.2374	
			4.44949	0.31201	–2.45833	23.12152	24.69444	25.00000	108.1244	101.2374	
			6.46410	0.42394	–2.45833	22.75739	24.69444	25.00000	109.8544	101.2374	
			8.47214	0.53559	–2.45833	22.54470	24.69444	25.00000	110.8908	101.2374	
			10.47723	0.64685	–2.45833	22.48219	24.69444	25.00000	111.1991	101.2374	
8.0	1	8.0	0.00000	0.02813	—	63.59564	—	64.00000	100.6358	—	
			2.41421	0.15531	–3.97222	62.04973	63.63272	64.00000	103.1431	100.5772	
			4.44949	0.20595	–3.97222	61.53839	63.63272	64.00000	104.0001	100.5772	
			6.46410	0.28058	–3.97222	60.85297	63.63272	64.00000	105.1715	100.5772	
			8.47214	0.35510	–3.97222	60.27128	63.63272	64.00000	106.1866	100.5772	
			10.47723	0.42916	–3.97222	59.79182	63.63272	64.00000	107.0380	100.5772	

Table A.2

$n_1 = n_2 = 2,\ n_3 = 4,\ \alpha_1 = \alpha_2 = \alpha_p = 0.50$

θ_{31}	σ_1^2	σ_3^2	λ_2	BIAS		MSE		MSE (V_3)	REL. EFF.	
				V	V*	V	V*	V_3	e(V,V_3)%	e(V*,V_3)%
1.0	1	1.0	0.00000	0.07105	—	0.46792	—	0.50000	106.8568	—
			2.41421	0.57080	–0.80893	0.90480	0.91646	0.50000	55.8782	54.5576
			6.46410	0.95447	–0.80893	2.16689	0.91646	0.50000	23.0746	54.5576
			8.47214	1.20548	–0.80893	3.22963	0.91646	0.50000	15.4816	54.5576
			10.47723	1.45608	–0.80893	4.54222	0.91646	0.50000	11.0078	54.5576
1.5	1	1.5	0.00000	0.03500	—	1.07764	—	1.12500	104.3953	—
			2.41421	0.46256	–1.23222	1.06001	2.05108	1.12500	106.1266	54.8493
			4.44949	0.51183	–1.23222	1.39599	2.05108	1.12500	80.5879	54.8493
			6.46410	0.70190	–1.23222	1.88664	2.05108	1.12500	59.6298	54.8493
			8.47214	0.89290	–1.23222	2.56320	2.05108	1.12500	43.8904	54.8493
			10.47723	1.08356	–1.23222	3.42981	2.05108	1.12500	32.8007	54.8493
2.0	1	2.0	0.00000	0.01971	—	1.93702	—	2.00000	103.2517	—
			2.41421	0.35312	–1.65317	1.64489	3.64808	2.00000	121.5891	54.8223
			4.44949	0.38646	–1.65317	1.91321	3.64808	2.00000	104.5366	54.8223
			6.46410	0.53712	–1.65317	2.18070	3.64808	2.00000	91.7138	54.8223
			8.47214	0.68732	–1.65317	2.59840	3.64808	2.00000	76.9704	54.8223
			10.47723	0.83723	–1.65317	3.16543	3.64808	2.00000	63.1826	54.8223

Contd ...

3.0	1	3.0	0.00000	0.00389	—	4.44177	—	4.50000	101.3109	—
			2.41421	0.21254	–2.49149	3.94497	8.22428	4.50000	114.0693	54.7161
			4.44949	0.24268	–2.49149	4.03588	8.22428	4.50000	111.4998	54.7161
			6.46410	0.34282	–2.49149	4.04804	8.22428	4.50000	111.1648	54.7161
			8.47214	0.44270	–2.49149	4.16069	8.22428	4.50000	108.1551	54.7161
			10.47723	0.54236	–2.49149	3.37214	8.22428	4.50000	102.9245	54.7161
5.0	1	5.0	0.00000	–0.00474	—	12.48616	—	12.50000	100.1108	—
			2.41421	0.10431	–4.16242	11.83734	22.88842	12.50000	105.5981	54.6128
			4.44949	0.12040	–4.16242	11.82675	22.88842	12.50000	105.6926	54.6128
			6.46410	0.17378	–4.16242	11.64191	22.88842	12.50000	107.3707	54.6128
			8.47214	0.22714	–4.16242	11.51152	22.88842	12.50000	108.5869	54.6128
			10.47723	0.28028	–4.16242	11.43196	22.88842	12.50000	109.3426	54.6128
8.0	1	8.0	0.00000	–0.00534	—	32.03244	—	32.00000	99.8987	—
			2.41421	0.04928	–6.66463	31.39770	58.64140	32.00000	101.9183	54.5690
			6.46410	0.08449	–6.66463	31.09911	58.64140	32.00000	102.8968	54.5690
			8.47214	0.11168	–6.66463	30.88126	58.64140	32.00000	103.6227	54.5690
			10.47723	0.13861	–6.66463	30.68402	58.64140	32.00000	104.2888	54.5690

Table A.3

$n_1 = n_2 = 10, n_3 = 2, \alpha_1 = \alpha_2 = \alpha_p = 0.50$

θ_{31}	σ_1^2	σ_3^2	λ_2	BIAS		MSE		MSE (V_3)	REL.EFF.	
				V	V*	V	V*	V_3	e(V, V_3)%	e(V*, V_3)%
1.0	1	1.0	0.00000	0.32376	—	0.79700	—	1.00000	125.4699	—
			3.44949	0.48015	–0.99816	0.87251	0.99977	1.00000	114.6119	100.0230
			5.74166	0.58120	–0.99816	0.99037	0.99977	1.00000	100.9720	100.0230
1.5	1	1.5	0.00000	0.23792	—	1.87034	—	2.25000	120.2988	—
			3.44949	0.35583	–1.49758	1.80982	2.24951	2.25000	124.3217	100.0218
			5.74166	0.43035	–1.49758	1.81637	2.24951	2.25000	123.8734	100.0218
2.0	1	2.0	0.00000	0.18790	—	3.51625	—	4.00000	113.7576	—
			3.44949	0.28235	–1.99696	3.37131	3.99935	4.00000	118.6481	100.0163
			5.74166	0.34026	–1.99696	3.30594	3.99935	4.00000	120.9944	100.0163
3.0	1	3.0	0.00000	0.13229	—	8.40013	—	9.00000	107.1412	—
			3.44949	0.19988	–2.99564	8.15347	8.99917	9.00000	110.3824	100.0093
			5.74166	0.23778	–2.99564	7.99303	8.99917	9.00000	112.5981	100.0093
5.0	1	5.0	0.00000	0.08340	—	24.29978	—	25.00000	102.8816	—
			3.44949	0.12674	–4.99294	23.94211	24.99900	25.00000	104.4185	100.0040
			5.74166	0.14412	–4.99294	23.64744	24.99900	25.00000	105.7197	100.0040
8.0	1	8.0	0.00000	0.05420	–	63.24638	—	64.00000	101.1916	—
			3.44949	0.08290	–7.98881	62.77304	63.99889	64.00000	101.9546	100.0017
			5.74166	0.08382	–7.98881	62.27390	63.99889	64.00000	102.7718	100.0017

REFERENCES

[1] Ali, M.A. and S.R. Srivastava., 1983. On power function of a sometimes pool test procedure in a mixed model-I : A theoretical investiagation. *Jour. Ind. Soci. Ag. Statistics*, 35, 80-90.

[2] Bancroft, T.A. and C.P. Han., 1977. Inference based on conditional specification : a note and a bibliography. *International Statistical Review*, **45**, 117-127.

[3] Han. C.P., C.V. Rao and J. Ravichandran., 1988. Inference based on conditional specification : A second bibliography. *Communication in Statistics - Theory and Methods.* **17** (6), 1945-1964.

[4] Singh, A.K., H.R. Singh and M.A. Ali., 1993. Error estimation in a mixed ANOVA model using two preliminary tests of significance. *Jour. Ind. Soci. Ag. Statistics.* 45(3). 372-388.

[5] Steel, R.G.D. and J.H. Torrie., 1980. Principles and Procedures of Statistics. Mc-Graw Hill Book Company, Inc. New York.

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Almost Separation of Bias Precipitates in the Estimator of 'Inverse of Population Mean' with Known Coefficient of Variation

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SUMMARY

The paper deals with the problem of estimating 'inverse of population mean' when coefficient of variation is known. A funnel connected with a filter-paper to filter the bias precipitate appearing in the estimators of the inverse of population mean is defined.

Key words : Bias precipitates, Linear variety of estimators, Mean square error, Coefficient of variation, Normal parent.

Introduction and Notations

In various investigations, the coefficient of variation shows stability and its value may be known accurately. The use of coefficient of variation as a priori has been made at a great length in the estimation of mean by several authors including Searls [4], Khan [3], Govindarajulu and Sahai [2] Gleser and Healy [1], Singh [7] [8], among others. Sen and Gerig [5], Sen [6] and Upadhyaya and Singh [14] have used the population shape parameters such as coefficient of skewness and kurtosis as apriori in addition to coefficient of variation in estimating the population mean.

The problem of estimation of the inverse of population mean arises in many situations, for instance, in Econometrics and Biological sciences ; see Zellner [15]. The conventional estimator of the inverse of population mean is the 'inverse of sample mean'. Improvements over the conventional estimator have been made by Srivastava and Bhatnagar [13] and Singh [9] in the situations, where population variance is known and unknown. Singh et al [11] have also improvements over conventional estimator of inverse of population mean using a priori information on shape parameters of population such as coefficient of skewness and kurtosis in addition to coefficient of variation.

A method adopted by Singh and Singh [12] to filter the bias precipitates from the estimators of inverse of population mean by using a funnel associated with a filter-paper is given. The apparatus consists of a linear variety of estimators and linear constraints. It would be seen that the chemicals (statistical constants) used for bias separation depend on the shape parameters of population and coefficient of variation. However, in case of normal population the reactants